# THERMAL SIGNATURE OF THE MAJORANA FERMION IN A JOSEPHSON JUNCTION

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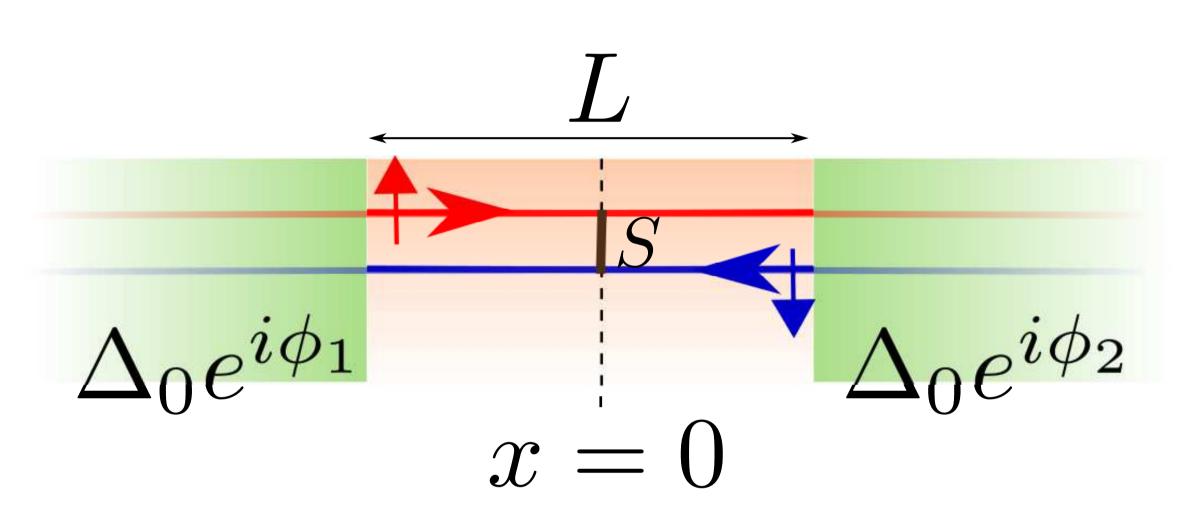
#### IDEA

Distinctive correlation between the thermal conductance and the Josephson current across a 1-D topological Josephson junction (T-JJ) is identified. A three terminal geometry is shown to be an ideal host to access these distinctive correlations which carries the hallmark of Majorana bound states (MBS) localized at the junction. We evaluate the thermal noise generated at the junction and show that they do not overwhelm our predictions.

### TWO-TERMINAL MAJORANA JJ

JJ based on HES of QSH state.

$$\mathcal{H} = (v_F \hat{p}_x \sigma_z - \mu) \tau_z + \Delta(x) (\cos \phi_r \tau_x - \sin \phi_r \tau_y)$$



 $S^T = S$ 

**Assumptions:** 

$$\mu >> \Delta_{
m o}$$

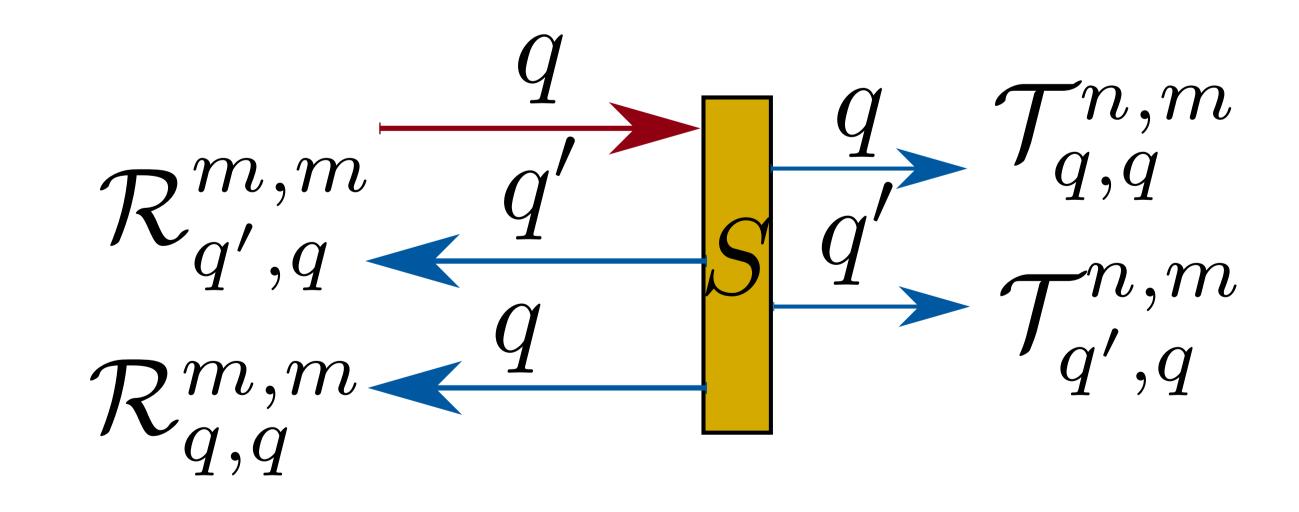
 $L<<\xi(=\hbar v_F/\Delta_{
m o})$ 

ABS energies:

ABS energies: Josephson Current: 
$$\omega_{\rm o}^{\pm} = \pm \Delta_{\rm o} \sqrt{\tau} \cos(\phi/2)$$
 
$$I^{J} = -\frac{2e\partial|\omega_{\rm o}^{\pm}|}{\tau}$$

Heat current:

$$I_n^H = \sum_{m 
eq n} \kappa_{n,m} (T_n - T_m)$$

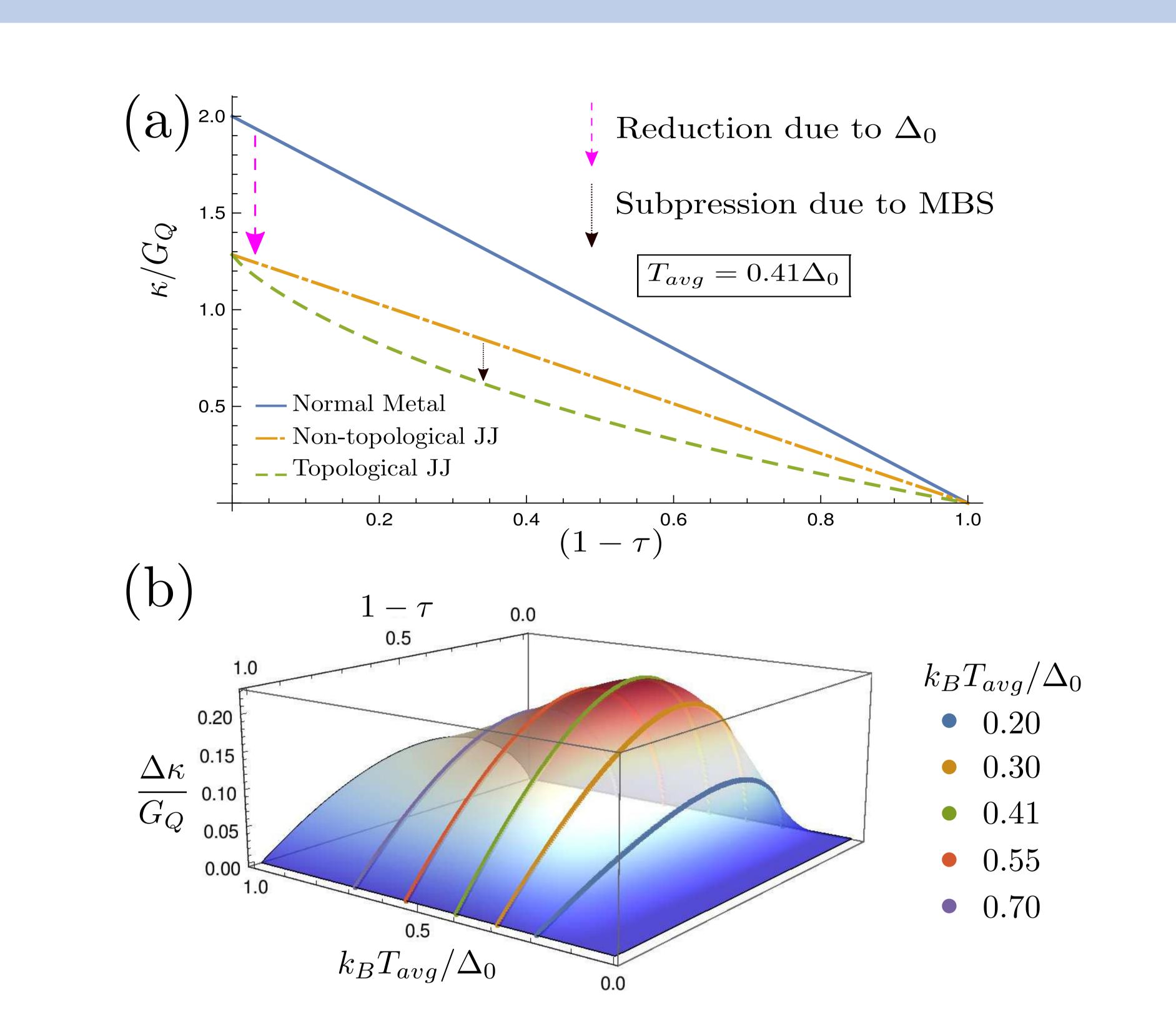


 $\mathcal{T}^{n,m} = \mathcal{T}_e^{n,m} + \mathcal{T}_h^{n,m} = (\mathcal{T}_{e,e}^{n,m} + \mathcal{T}_{h,e}^{n,m}) + (\mathcal{T}_{h,h}^{n,m} + \mathcal{T}_{e,h}^{n,m})$ 

Thermal conductance:

$$\kappa_{n,m}(\phi_j) = \left[ rac{1}{h} \int_{\Delta_0}^{\infty} d\omega \, \omega \, \{\mathcal{T}^{n,m}\} rac{df(\omega,T)}{dT} 
ight]_{T=T_{ ext{avg}}}$$

For Majorana JJ,  $\phi$  and  $\tau$  dependence of  $\mathcal T$  enters only via the dependence of  $\omega_0^\pm$  appearing in the denominator (apart from overall multiplicative dependence of  $\tau$ ).



(a)  $\kappa/G_0$  for  $\phi = 0$  as a function of  $\tau$  for a normal junction and the corresponding non-topological and topological JJ. (b) 3-D surface plot of difference between topological and non-topological case in the  $(1-\tau)$ - $(k_BT_{\rm avg}/\Delta_0)$  plane.

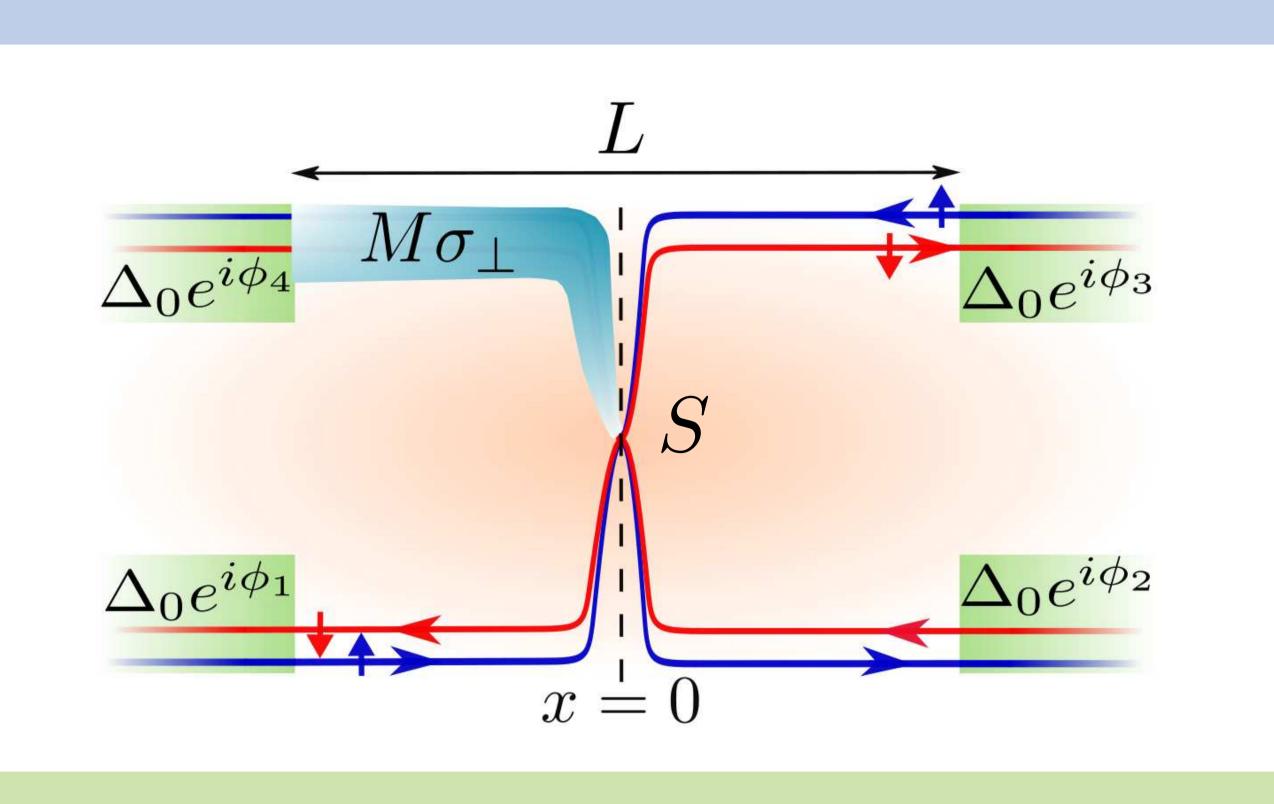
LDS of BdG QP just above the gap.

$$\kappa = \sigma^N \mathcal{I}(T_{\text{avg}}, \Delta_{\text{o}}, \tau, \phi)$$
 $\partial_{\phi} \kappa = \sigma^N I^J \mathcal{J}(T_{\text{avg}}, \Delta_{\text{o}}, \tau, \phi)$ 

The quantities  $\kappa$  and  $\partial_{\phi}\kappa$  are dominated by the poles at the ABS energies which for Majorana JJ makes the integrals  $\mathcal{I}$ and  $\mathcal J$  universal.

## THREE-TERMINAL MAJORANA JJ

$$egin{aligned} \mathcal{H} &= H_{ ext{edge}} + H_{ ext{Zeeman}} \ &= \{(\eta v_F \hat{p}_x \sigma_z - \mu) au_z + \Delta(x) (\cos \phi_r au_x - \sin \phi_r au_y)\} \ &+ \{(\eta - extbf{1})/2\} M_x \sigma_ot \end{aligned}$$

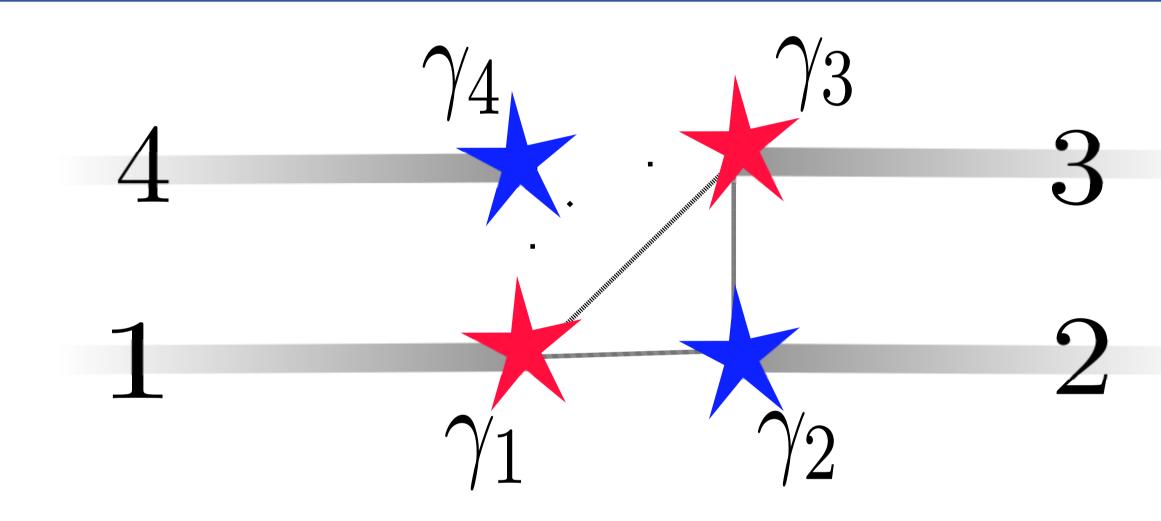


ABS energies:

$$\omega_0^0 = 0$$

$$\omega_0^{\pm} = \pm \Delta_0 \sqrt{\tau_{12} \cos^2 \frac{\phi_{12}}{2} + \tau_{13} \sin^2 \frac{\phi_{13}}{2} + \tau_{23} \cos^2 \frac{\phi_{23}}{2}}$$

### EFFECTIVE MAJORANA MODEL



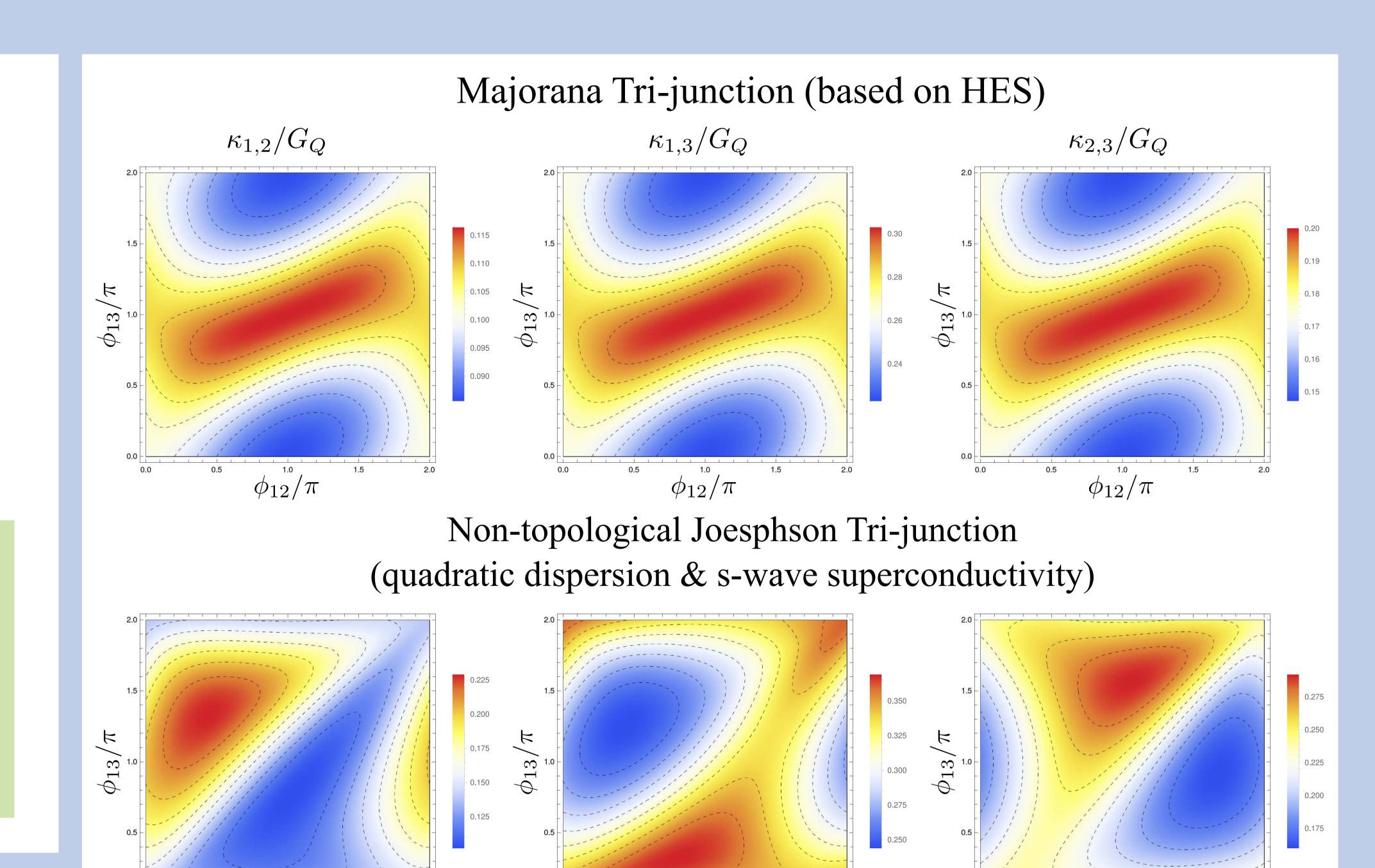
$$H=rac{i}{2}\sum_{1\leq a\leq b\leq 4}\xi_{ab}\gamma_a\gamma_b \ \xi_{ab}=rac{\Delta_0}{2}\sqrt{ au_{ab}}\cos\left(rac{\phi_{ab}}{2}-rac{\chi_{ab}}{2}
ight)$$

 $t_{i_4} = t_{4i} = 0 \text{ for } i \neq 4 \Rightarrow \xi_{14} = \xi_{24} = \xi_{34} = 0$  $c_{+} = (\gamma_{1} + i\gamma_{2})/2$  and  $c_{-} = (\gamma_{3} + i\gamma_{4})/2$ 

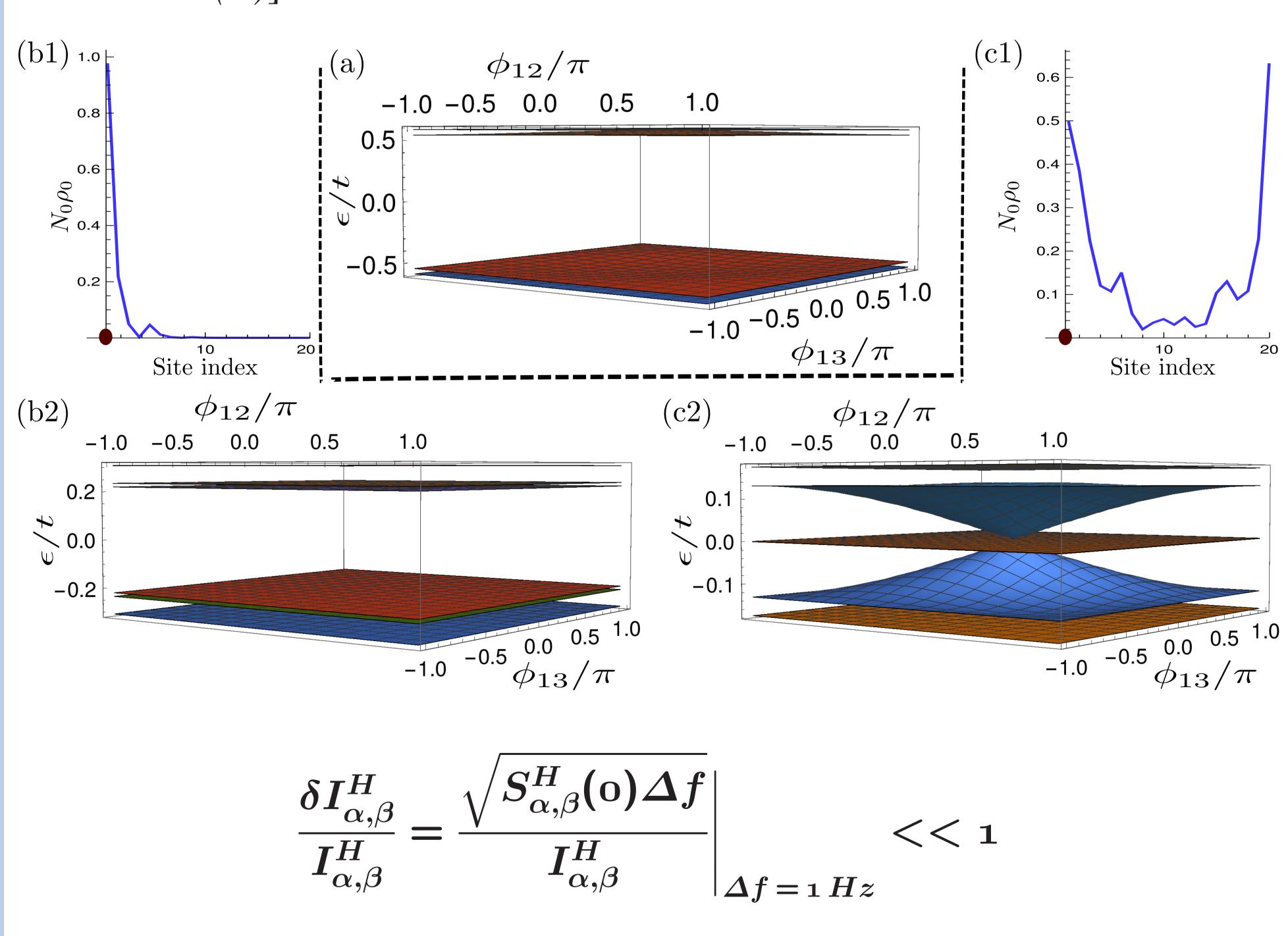
Hybridization of any odd number of Majoranas always gives one Majorana, stuck at zero energy.

### RESULTS

$$egin{aligned} rac{\kappa_{i,j}}{\kappa_{k,l}} = rac{\sigma_{i,j}^N}{\sigma_{k,l}^N} \ rac{\partial_{\phi_{kl}} \kappa_{i,j}/I_{k,l}^J}{\partial_{\phi_{pq}} \kappa_{m,n}/I_{p,q}^J} = rac{\sigma_i^I}{\sigma_m^N} \end{aligned}$$



- ► True for JJ with quadratic dispersion with p-wave superconductivity.
- ➤ Violations: finite junctions & non-topological Josephson tri-junctions.
- ► Topological 'Andreev type end states' cannot mimic[*Phys. Rev. B 100*, 220502(R)].



#### SUMMARY

Our approach combines information form sub-gap Cooper pair transport and transport of above-gap thermally excited Bogoliubov QP to identify the signatures of Majorana fermions which is complementary to the existing studies.

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