Finding exotic superconducting pairing in topological semimetals

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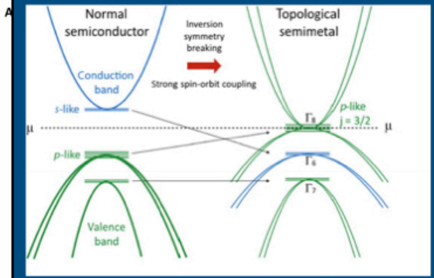
Properties of the spin-orbit coupled semimetals



Sci. Adv. 4, 4513 (2018)

1. Strong spin-orbit interaction leading to band inversion similar to topological insulator
2. Superconducting ground state with higher-spin Fermi surface: Pairing of spin-3/2 electrons

PRL 116, 177001 (2016)
PRB 98, 104514 (2018)

Symmetry operators for spin=3/2

$$\mathcal{J}\Delta_{m_1,m_2}(r_1,t_1;r_2,t_2)=\Delta_{m_2,m_1}(r_1,t_1;r_2,t_2)$$
$$\mathcal{P}\Delta_{m_1,m_2}(r_1,t_1;r_2,t_2)=\Delta_{m_1,m_2}(r_2,t_1;r_1,t_2)$$
$$\mathcal{T}\Delta_{m_1,m_2}(r_1,t_1;r_2,t_2)=\Delta_{m_1,m_2}(r_1,t_2;r_2,t_1)$$

We introduce the symmetry classification

$$\mathcal{JPT}\Delta_{m_1,m_2}(r_1,t_1;r_2,t_2)=-\Delta_{m_1,m_2}(r_1,t_1;r_2,t_2)$$

TABLE I. Pairing symmetry for $j=3/2$.

Pairing state $\lvert j,m\rangle$	Ang. mom. ($\mathcal{J}$)	Parity ($\mathcal{P}$)	Freq./Time ($\mathcal{T}$)
Singlet $j=0;\ m=0$	Odd	Even	Even
		Odd	**Odd**
Triplet $j=1;\ -1\le m\le 1$	Even	Even	**Odd**
		Odd	Even
Quintet $j=2;\ -2\le m\le 2$	Odd	Even	Even
		Odd	**Odd**
Septet $j=3;\ -2\le m\le 3$	Even	Even	**Odd**
		Odd	Even

When two spin-3/2 electrons couple

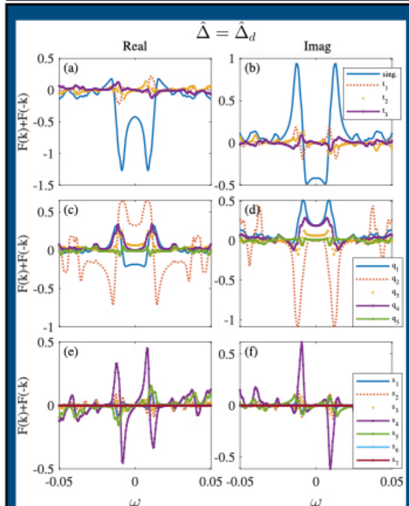
$$l_i=1, s_i=1/2, j_i=3/2$$

- Total angular momentum: $|j_1-j_2|<j<j_1+j_2$

$$|j,m\rangle=\left(\sum_{m_1=-j_1}^{j_1}\sum_{m_2=-j_2}^{j_2}|j_1,j_2;m_1,m_2\rangle\langle j_1,j_2;m_1,m_2|\right)|j,m\rangle$$

$$\sum_{m_1,m_2}\underbrace{\langle j_1,j_2;m_1,m_2|j,m\rangle}_{\text{C.G. coefficient}}|j_1,j_2;m_1,m_2\rangle \;\downarrow\; |m_1,m_2\rangle$$

- Singlet state (j=0) $|0,0\rangle$
- Triplet state (j=1) $|1,-1\rangle\ |1,0\rangle\ |1,1\rangle$
- Quintet state (j=2) $|2,-2\rangle\ |2,-1\rangle\ |2,0\rangle\ |2,1\rangle\ |2,2\rangle$
- Septet state (j=3) $|3,-3\rangle\ |3,-2\rangle\ |3,-1\rangle\ |3,0\rangle\ |3,1\rangle\ |3,2\rangle\ |3,3\rangle$



Pair amplitude (PA)

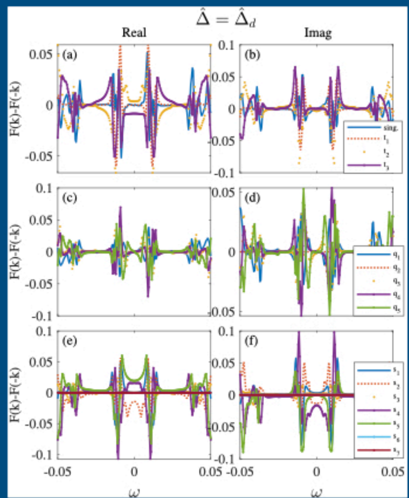
$$\mathcal{F}_{m_1,m_2}(r_1,t_1;r_2,t_2)=\langle\mathcal{T}\Psi_{m_1}(r_1,t_1)\Psi_{m_2}(r_2,t_2)\rangle$$

- Even-parity PA

$$\mathcal{F}(\boldsymbol{k})+\mathcal{F}(-\boldsymbol{k})$$

- Odd-parity PA

$$\mathcal{F}(\boldsymbol{k})-\mathcal{F}(\boldsymbol{k})$$



Hamiltonian for a cubic material

$$H_0(k)=\underbrace{\alpha k^2}_{\text{kinetic energy}}+\beta\sum_\nu k_\nu^2J_\nu^2+\gamma\sum_{\nu\neq\nu'}k_\nu k_{\nu'}J_\nu J_{\nu'}-\tilde{\mu}$$

kinetic energy ↓ spin-orbit interaction ↓ chemical potential

BdG Hamiltonian

$$\check{\mathcal{H}}_k=\begin{pmatrix}\hat{H}_0(\boldsymbol{k}) & \hat{\Delta}(\boldsymbol{k})\\ \hat{\Delta}^\dagger(\boldsymbol{k}) & -\hat{H}_0^T(-\boldsymbol{k})\end{pmatrix}$$

$$\Delta(\boldsymbol{k})=\Delta_1\psi_k\eta_s+\Delta_0(\eta_{xz}+i\eta_{yz})$$

$$\eta_{ij}=\frac{1}{\sqrt{3}}(\hat{J}_j\hat{J}_i+\hat{J}_i\hat{J}_j)U_T$$

Summary:

- **Introducing symmetry classification for spin-3/2**
- Total 32 possible pairings (including even- and odd-frequency)
- Finite pair amplitudes for various exotic pair symmetries

Reference: Dutta, Parhizgar, BlackSchaffer, arXiv: very soon

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