Many-body Majorana-like zero modes without gauge symmetry breaking

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Introduction

Topological superconductors represent one of the key hosts of Majorana-based topological quantum computing. Typical scenarios for one-dimensional topological superconductivity assume a broken gauge symmetry associated to a superconducting state. In this work, we demonstrate that robust zero modes appear in a 1D many-body model without gauge symmetry breaking. The model we focus on would give rise to a topological superconductor at the mean-field level if the gauge symmetry were explicitly broken. We demonstrate that no such gauge symmetry breaking is required for the emergence of Majorana-like zero modes, establishing a peculiar paradigm of quantum many-body excitations with no single-particle analog. Despite their fundamental differences to Majorana zero modes, we demonstrate that these two types of many-body excitations share many properties, including robustness to perturbations and disorder.

Kitaev model

Kitaev model is a chain of spinless fermions with p-wave superconducting pairing between the neighboring sites:

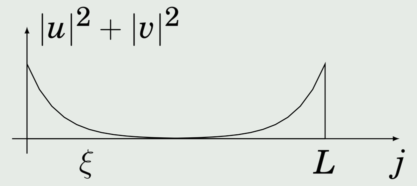
$$\hat{H}_K = -t\sum_{j=1}^{L-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1}\right) + \Delta\sum_{j=1}^{L-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_j^\dagger + \hat{a}_j \hat{a}_{j+1}\right) - \mu\sum_{j=1}^{L}\hat{a}_j^\dagger \hat{a}_j$$

This Hamiltonian can be diagonalized by a linear Bogoliubov transformation:

$$\hat{a}_j = \sum_k\left(u_{jk}\hat{c}_k + v_{jk}^*\hat{c}_k^\dagger\right),\ \hat{a}_j^\dagger = \sum_k\left(v_{jk}\hat{c}_k^\dagger + u_{jk}^*\hat{c}_k\right),\quad \hat{H}_K = \varepsilon_0\hat{c}_0^\dagger\hat{c}_0 + \sum_{j=1}^{L-1}\varepsilon_k\hat{c}_k^\dagger\hat{c}_k + const$$

$\varepsilon_0 \propto \Delta e^{-L/\xi}$, $\varepsilon_k > \Delta$ $(k > 1)$

One could use the states $|\Psi_0\rangle$ and $\hat{c}_0^\dagger|\Psi_0\rangle$ as a qubit!

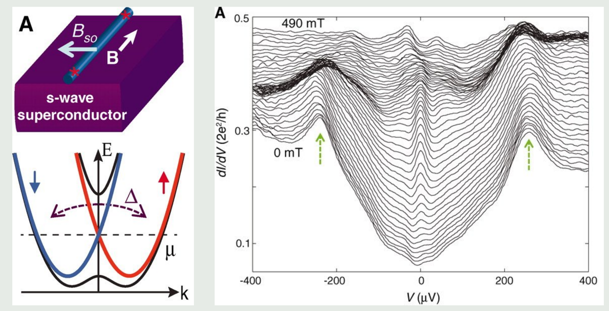


A. Y. Kitaev, Unpaired majorana fermions in quantum wires, Physics-Uspekhi 44, 131–136 (2001).

Experimental implementations

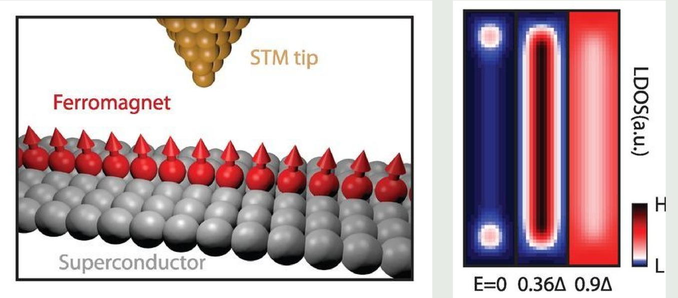
- One-dimensional channel
- Rashba spin-orbit coupling
- Strong Zeeman field
- s-wave superconductivity

V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, *Science* **336**, 1003 (2012).



- Chain of magnetic atoms on a superconductor surface

S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. A. Bernevig, and A. Yazdani, *Science* **346**, 602



A mean field superconductivity is required for all these realizations. But is it possible to have Majoranas without gauge symmetry breaking?

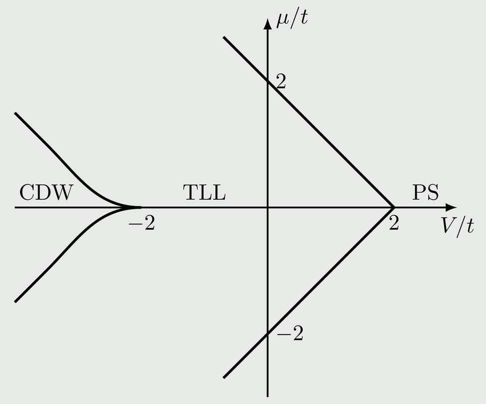
Interacting model

We consider Kitaev model and replace superconducting pairing term with an attractive interaction between the neighboring sites. Treated within a mean-field approximation such interaction gives rise to p-wave superconductivity.

$$\hat{H}_I = -t\sum_{j=1}^{L-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1}\right) - V\sum_{j=1}^{L-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_{j+1} - \frac{1}{2}\right)\left(\hat{a}_j^\dagger \hat{a}_j - \frac{1}{2}\right) - \mu\sum_{j=1}^{L}\hat{a}_j^\dagger \hat{a}_j$$

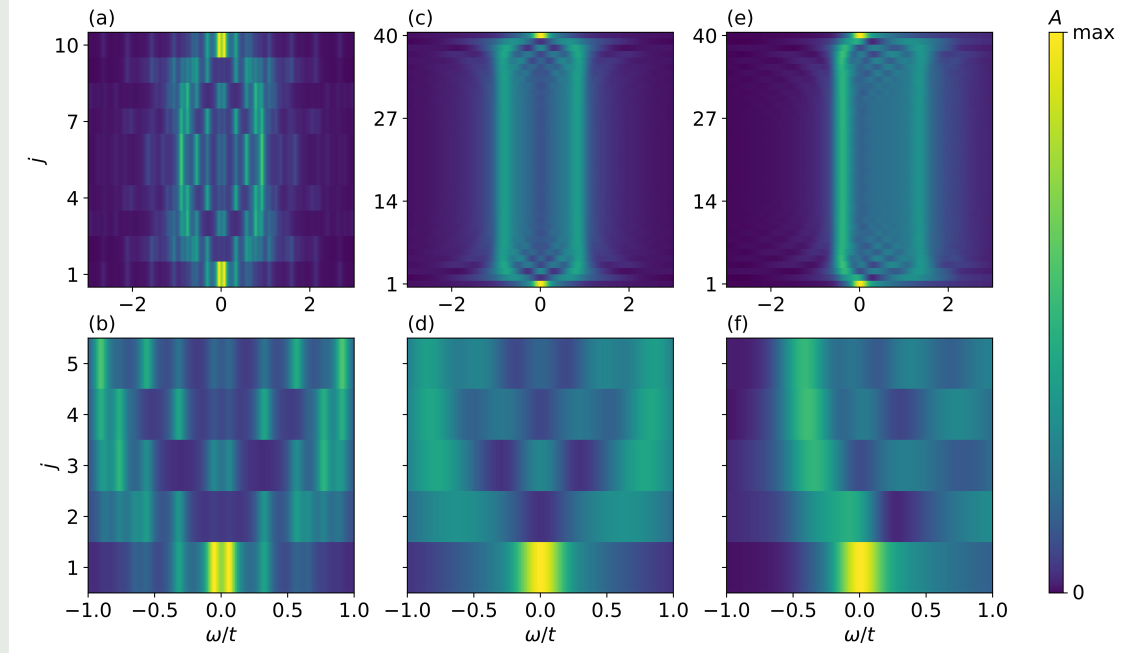
Properties of model:

- Gauge invariant
- Bethe-ansatz integrable
- Yields Kitaev model in a mean field approximation
- Has three phases: phase separation (PS), Tomonaga–Luttinger liquid (TLL), and charge density wave (CDW)
- Gapless in the TLL phase



Spectral function

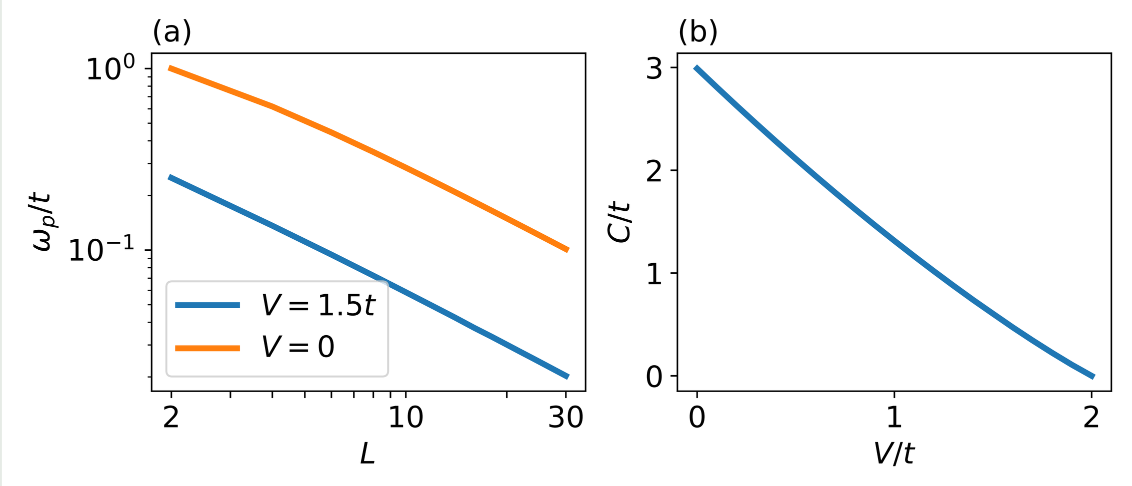
$$A(j,\omega) = \langle\Psi_0|\hat{a}_j\delta(\omega - \hat{H} + E_0)\hat{a}_j^\dagger + \hat{a}_j^\dagger\delta(\omega + \hat{H} - E_0)\hat{a}_j|\Psi_0\rangle = \sum_m\left[|\langle\Psi_0|\hat{a}_j|\Psi_m\rangle|^2\delta(\omega - E_m + E_0) + \langle\Psi_0|\hat{a}_j^\dagger|\Psi_m\rangle|^2\delta(\omega - E_0 + E_m)\right]$$



(a, b) $L = 10$, $\mu = 0$, $V = 1.5t$ **(c, d)** $L = 40$, $\mu = 0$, $V = 1.5t$ **(e, f)** $L = 40$, $\mu = -0.2t$, $V = 1.5t$

Peak splitting

$$\omega_p = \frac{E_0(N_0+1) + E_0(N_0-1)}{2} - E_0 \propto \frac{C(V)}{L}$$



The peak splitting vs. the chain length and dependence of the prefactor C on the interaction strength at half filling $\mu = 0$.

Robustness to the perturbations

(a, b) Second neighbor hopping

$$\hat{H}_{2nh} = -t'\sum_{j=1}^{L-2}\left(\hat{a}_{j+2}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+2}\right)$$

$L = 40$, $\mu = 0$, $V = 1.5t$, $t' = -0.2t$

(c, d) Second neighbor interaction

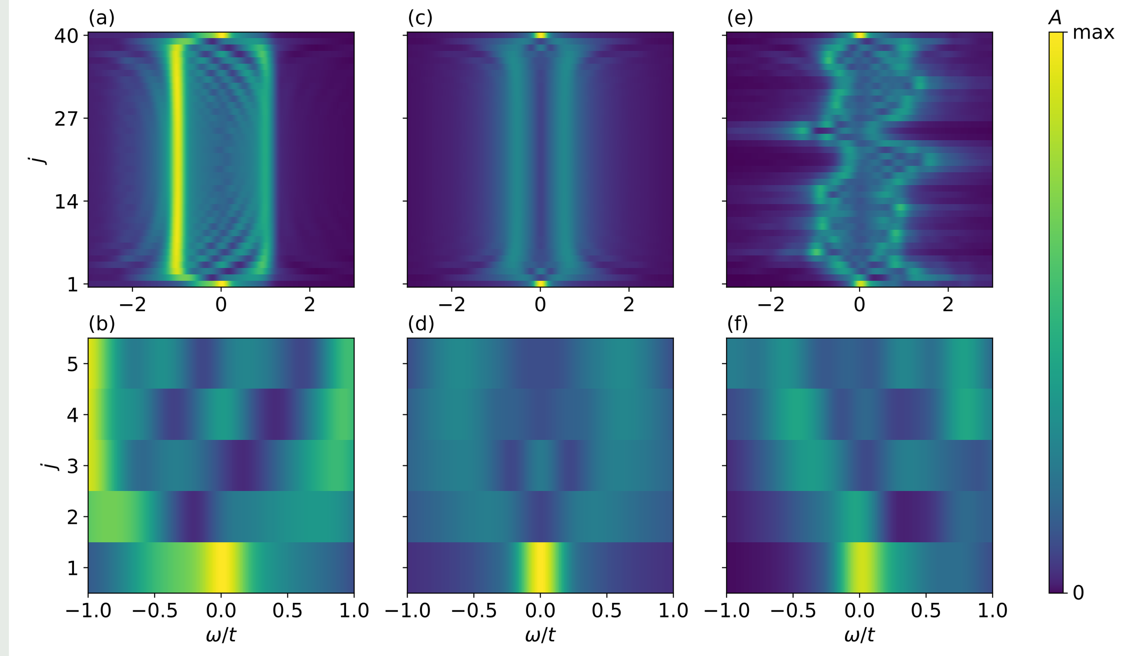
$$\hat{H}_{2ni} = -V'\sum_{j=1}^{L-2}\left(\hat{a}_{j+2}^\dagger \hat{a}_{j+2} - \frac{1}{2}\right)\left(\hat{a}_j^\dagger \hat{a}_j - \frac{1}{2}\right)$$

$L = 40$, $\mu = 0$, $V = 1.5t$, $V' = 0.4t$

(e, f) On-site disorder

$$\hat{H}_d = \sum_{j=1}^{L} u_j\hat{a}_j^\dagger\hat{a}_j,\ u_j \in [-W, W]$$

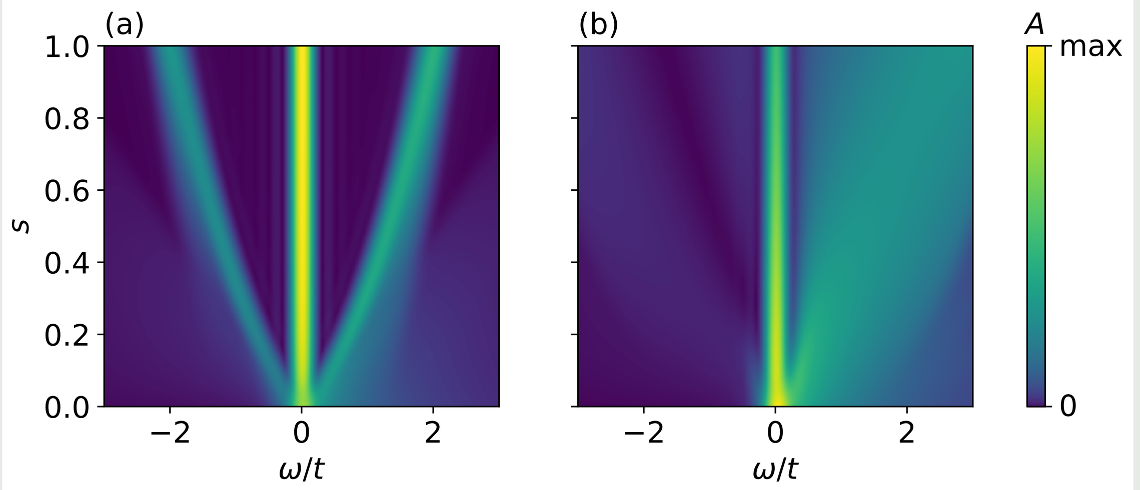
$L = 40$, $\mu = 0$, $V = 1.5t$, $W = 0.5t$.



Connection to a topological superconductor

Parametric Hamiltonian

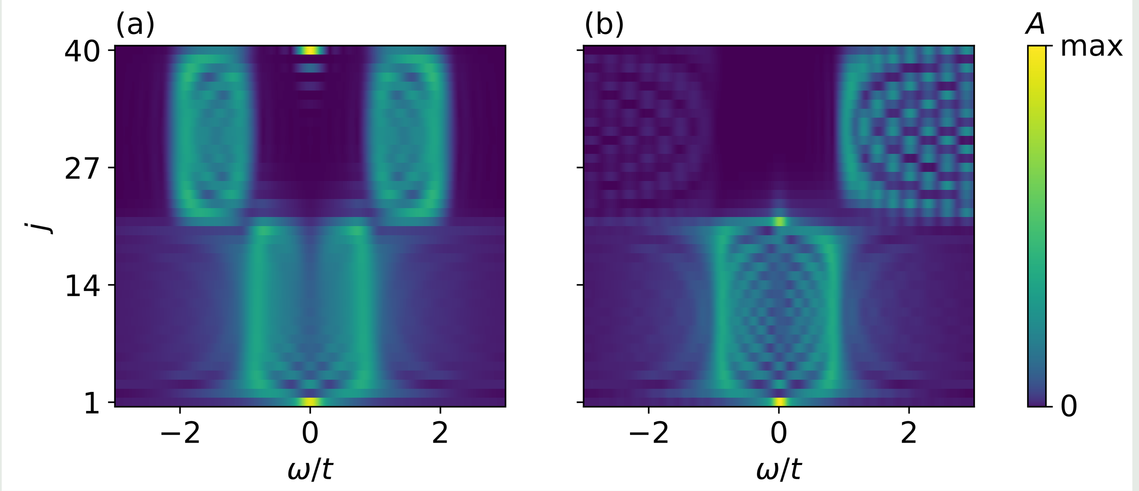
$$\hat{H}(s) = (1-s)\hat{H}_I + s\hat{H}_K - \tilde{\mu}(s)\hat{N},\quad 0 \leqslant s \leqslant 1$$



Local density of states at the first site **(a)** $L = 40$, $V = 1.5t$, $\Delta = t$, $\mu = -0.2t$, $\tilde{\mu} = 0$ **(b)** $L = 40$, $V = 1.5t$, $\Delta = t$, $\mu = -0.2t$, $\langle\hat{N}\rangle = L/4$

Interface between the interacting model and the topological superconductor

$$\hat{H} = -t\sum_{j=1}^{L-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_{j+1}\right) - V\sum_{j=1}^{L'-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_{j+1} - \frac{1}{2}\right)\left(\hat{a}_j^\dagger \hat{a}_j - \frac{1}{2}\right) + \Delta\sum_{j=L'}^{L-1}\left(\hat{a}_{j+1}^\dagger \hat{a}_j^\dagger + \hat{a}_j \hat{a}_{j+1}\right) - \mu\sum_{j=1}^{L'}\hat{a}_j^\dagger \hat{a}_j - \mu'\sum_{j=L'+1}^{L}\hat{a}_j^\dagger \hat{a}_j$$



Density of states for all sites for the interface with topological superconductor **(a)** $L = 40$, $L' = 20$, $V = 1.5t$, $\Delta = 0.5t$, $\mu = 0$, $\mu' = 0$ **(b)** $L = 40$, $L' = 20$, $V = 1.5t$, $\Delta = 0.5t$, $\mu = 0$, $\mu' = 3t$

Continuous limit

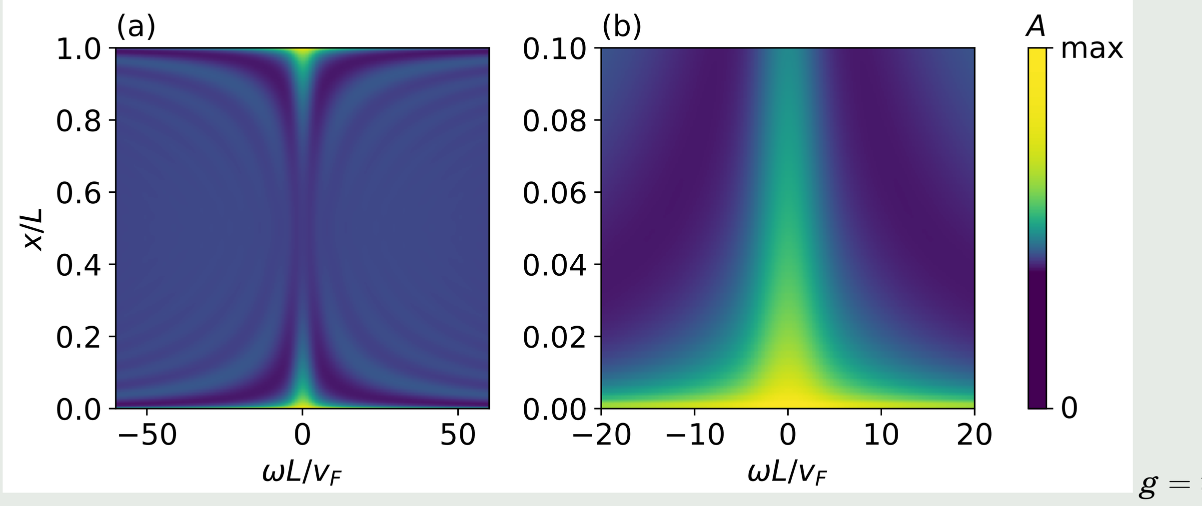
Continuous wire

$$\hat{H}_c = \int_0^L\left\{\hat{\psi}^\dagger\left(-\frac{\partial_x^2}{2m} - \frac{k_F^2}{2m}\right)\hat{\psi} + \frac{g}{8k_F^2} : [\partial_x\hat{\rho}]^2 :\right\}\mathrm{d}x,\quad \hat{\psi}(0) = \hat{\psi}(L) = 0$$

Spectral function

$$A(x,\omega) = i\int_{-\infty}^{+\infty}\left[G^>(x,x,t) - G^<(x,x,t)\right]e^{i\omega t}\,\mathrm{d}t$$

$$G^>(x,x't) = -i\langle\hat{\psi}(x,t)\hat{\psi}^\dagger(x',0)\rangle\quad G^<(x,x't) = i\langle\hat{\psi}^\dagger(x',0)\hat{\psi}(x,t)\rangle$$



$g = v_F$

Conclusions

We have demonstrated the emergence of robust Majorana-like edge modes in the many-body quantum system without mean-field superconductivity.

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